PERIODIC INSPECTION FOR SAFETY OF CANDU HEAT TRANSPORT PIPING SYSTEMS - A PROBABILISTIC APPROACH

by

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SUMMARY

A rational approach is developed for prediction of failure risk or probability of survival of piping systems in a nuclear power plant. The effects of various inspection schemes on the risk of failure are then investigated. The stated objective of this phase of the proposed R & D Programme was to formulate a probabilistic model to predict failure risk at a given time. To apply the proposed method to real situations in a nuclear power plant, a list of input data is specified. Through an example of a pressurized pipe containing a defect, it is clearly shown that the required data is easily obtainable. The formulation and demonstration of functional relationships will enable us to investigate the effects of periodic inspection on the safety of heat transport piping system and the strategy to maintain a threshold safety index throughout the plant life. Computer programmes have to be developed to find numerical values and conducted sensitivity analysis.

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SOMMAIRE

Dans cette étude, nous avons développé une approche rationnelle afin de prédire le risque de rupture ou la probabilité de survie (fiabilité) d'un système de tuyauterie dans une centrale nucléaire. L'influence de différents types de procédure d'inspection sur le risque de rupture est alors examinée. Le but de cette phase du programme de recherche et de développement est d'établir un modèle mathématique permettant de prédire le risque de ruine en fonction du temps. Une liste de données est spécifiée permettant l'application de la méthode proposée face aux situations réelles existant dans une centrale nucléaire. Par le biais de l'exemple d'un tuyau (vaisseau) sous pression contenant une fissure, nous indiquons que les données peuvent être facilement obtenues. La formulation proposée et la démonstration d'obtention des relations fonctionnelles permettront d'étudier les effets d'inspection périodique sur la sécurité du système de tuyauterie du caloporteur et la stratégie de maintenir un indice de sécurité de base durant la vie d'une centrale. Il est nécessaire de développer des programmes d'ordinateur afin de résoudre les équations présentées et analyser la sensibilité du modèle selon les différents paramètres.
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INTRODUCTION

The objective of this R and D Programme has been to develop a methodology and to provide a rational basis for periodic inspection, taking into consideration variability of various parameters. Periodic inspection is generally carried out for the purpose of ensuring an adequate level of safety throughout the life of a plant, and to protect plant personnel and the public from consequences of a failure and release of fission products. A broad view of periodic inspection philosophy was given in the introduction to the Phase I-a report of the present R & D Programme. To make this report to some extent self contained, we have reproduced it in the Appendix to this report.

It is generally agreed that there exists some degree of uncertainty in most phenomena encountered or observed in engineering practice. In mechanical systems, the loads (applied or transformed), the material strength to resist them and defect sizes, are the most common variables which exhibit scatter. In recent years there has been a considerable effort to deal with such situations by employing the theory of probability. Today, even legal codes such as Canadian National Building Code are given a rational probabilistic background.

Beginning with a probability-based design theory and eliminating all variability in the design parameters, the residual would be a special case, corresponding to the classical deterministic design.
The purpose of this report is to present theory and methodology in sufficient detail to predict risk of failure of pressure retaining components in nuclear power plants. The methodology proposed here is equally applicable to other mechanical systems and components. In order to avoid any conceptual difficulty in appreciating the notions of probability theory, the first part of this report is concerned with defining probability of failure, multiple parameter reliability and risk of failure functions. A probabilistic model for crack initiation and propagation is outlined in section 2. Analytical and experimental data are used to define the probability density function of fatigue life and life factor. A pressurized pipe containing a defect is then used as an example to derive median residual strength in terms of defect growth.

In the proposed model there are three possible failure modes during any given life cycle of a member of the population. The risk of failure for each mode is derived and the combined total risk is determined as a function of time (or number of load applications).

The effects of various types of inspection on the risk of failure or probability of survival is then investigated. Finally, it is shown that the input data for the application of the proposed method is rather easy to obtain. The previous example of a pressurized pipe in a nuclear plant is used to demonstrate the required functional relationships.
1. **RELIABILITY**

1.1 **Notion of Probability of Failure**

The resistance, $R$, of a component or structure associated with a specific type of applied load is not a deterministic value, it varies in a random fashion. A major reason for this type of behaviour is inhomogeneity of the material. Even if one takes special care to ensure a rather homogeneous material on a macroscopic scale, the orientation, size and defects in the crystalline structure and grain boundaries on the microscopic scale will result in a variable response to a given load.

It is therefore quite reasonable to assume that the strength of a test sample, component or structure, is of a statistical nature. It can then be represented by a statistical distribution function. Similarly, the applied load, $S$, is a statistical variable. The main reason in this case is the variability of the external agents which induce the load and the transfer mechanisms.

Failure is the event when $R < S$. The word "failure" is used here in a most general sense, and it implies any kind of operational deficiency or other hazard caused by the application of the loading resulting in the unserviceability of the component or structure. Let us denote by $f_S(s)$ and $f_R(r)$ the probability density function (PDF) of the load and resistance respectively,
and $F_S(s)$ and $F_R(r)$ their respective cumulative distribution function (CDF). We could also define a joint probability density function $f_{R,S}(r,s)$ which is a measure of probability of resistance $R$ and load $S$, simultaneously having values between $r$ and $r + dr$ and, $s$ and $s + ds$, respectively. The probability of failure, $p_f$, of a component or structure may be obtained by evaluating the integral:

$$p_f = \int_{\mathcal{R}} \int_{\mathcal{S}} f_{S,R}(s,r) \, dr \, ds \quad (1.1)$$

A schematic interpretation of the above equation is shown in Fig. 1.1.

If one assumes that $R$ and $S$ are both positive and statistically independent, then,

$$P[R \cap S] = P[R]P[S] \quad (1.2)$$

consequently,

$$p_f = \int_{\mathcal{R}} \int_{\mathcal{S}} f_S(s) f_R(r) \, dr \, ds \quad (1.3)$$

Figure 1.2 shows the schematic interpretation of Eq. (1.2).

Using the notion of cumulative density function, one may write (1.3) as:
Fig. 1.1: Schematic Interpretation of Probability Functions

Fig. 1.2: Probability of failure and statistical independence
Equation (1.4a) states that probability of failure is equal to the product of the probability that the resistance $R$ is less than the value of $s$, $F_R(s)$, and the probability that the load has a value between $s$ and $s + ds$, $f_S(s)$, and summing for all values of $s$. A similar interpretation can be given for the relation (1.4b).

It should be noted that the probability of survival, not the probability of failure, is a direct measure of safety of a structure subject to a random sequence of loads taken from the population $F_S(s)$ during its service life. The probability of survival is also referred to as the reliability function when interpreted as a function of time or its equivalent.

$$\text{Reliability} = L = P(R > S) = \int_{0}^{\infty} [1-F_R(s)] f_S(s) \, ds \quad (1.5)$$

$$= 1-p_f$$
1.2 Multiple Variable Functions

A component or a system may be subjected to various types of loads (e.g. static, transient, cyclic, etc). In such a case, the failure may occur in different modes. To calculate the probability of failure or reliability function, one has to extend the previous definition, (1.1 to 1.5) to multiple parameter functions.

Let a component or structure be subjected to n types of loads, \( S_1, S_2, \ldots, S_n \). Suppose the resistance to these loads be \( R_1, R_2, \ldots, R_q \). There will be a joint density function of n loads and q resistances of the form:

\[
f_{S_1, S_2, \ldots, S_n, R_1, R_2, \ldots, R_q}(s_1, s_2, \ldots, s_n, r_1, r_2, \ldots, r_q)
\]

One may now calculate the probability of failure from:

\[
P_f = \int_{R<s} \int_{R<q} f_{S_1, \ldots, R_q}(s_1, \ldots, r_q) ds_1 \ldots dr_q \quad (1.6)
\]

If one assumes that the \((n+q)\) variables are statistically independent, then the probability of failure becomes

\[
P_f = \int_{R<s} \int_{R<q} f_{S_1}(s_1)f_{S_2}(s_2) \ldots f_{R_q}(r_q) ds_1 \ldots dr_q \quad (1.7)
\]

The case where one of the load or resistance variables or both are dependent on some of the other parameters is
of particular interest in this study. Suppose that the load, \( S \), depends on a variable \( A \) and that the resistance \( R \) is dependent on another variable \( B \). Then there will exist a probability density function \( f_{R,S,A,B}(r,s,a,b) \). If there is a statistical independence between load and resistance, one can then write Eq. (1.6) in the following form:

\[
P_f = \iiint_{R<S} f_{S,A}(s,a)f_{R,B}(r,b)\,dbrdads \quad (1.8)
\]

Using the notion of conditional density, relation (1.8) could be written as

\[
P_f = \iiint_{R<S} f_{S|A}(s,a)f_{A}(a)f_{R|B}(r,b)f_{B}(b)\,dbrdads \quad (1.9)
\]

where \( f_{S|A}(s,a) \) represents the probability that the load \( S \) takes a value between \( s \) and \( s + ds \), given that the variable \( A \) has already taken a value \( a \).

1.3 Reliability Functions

The reliability of a component or structure may be calculated from Equation (1.5). However in practice, most components are subjected to varying types of loads with time, thus the reliability at a given time is of interest. In this case,
one employs a reliability function where the variable is time $T$ or some equivalent parameter, such as the number, $N$, of load applications. Thus, the reliability function is defined as

$$L_N(n) = P[N>n]$$ (1.10)

where $P[N>n]$ designates the probability of event $N>n$, i.e., that the number of cycles to failure, $N$, is greater than those applied, $n$. The probability of failure in the interval $[1,n]$ is given by

$$F_N(n) = 1 - L_N(n) = P[N \leq n]$$ (1.11)

Further definitions could be introduced such that $f_N(n)$ (probability mass function PMF) which denotes the probability that the structure will fail at exactly the $n^{th}$ application of load, i.e.,

$$f_N(n) = P[N=n] = F_N(n) - F_N(n-1)$$ (1.12)

Another definition frequently used is the risk of failure $h_N(n)$ denoting the probability that the structure which has survived $(n-1)$ applications of load will fail at the $n^{th}$ one, i.e.,

$$h_N(n) = P[N=n|N>n-1] = \frac{P[(N=n) \cap (N>n-1)]}{P[N>n-1]}$$ (1.13)
The above equation shows the relationship among \( f_N(n), h_N(n), L_N(n) \) and \( F_N(n) \). Functions \( f_N(n) \) and \( h_N(n) \) are also known as the mortality distribution and the failure rate, respectively.

If "n" is treated as a continuous random variable such as time \( T \), then one obtains

\[
F_T(t) = \int_0^t f_T(\tau) d\tau \quad (1.14)
\]

\[
L_T(t) = \int_t^\infty f_T(\tau) d\tau \quad (1.15)
\]

\[
h_T(t) = \frac{f_T(t)}{1 - F_T(t)} \quad (1.16)
\]

In situations where the probability of failure \( F_T(t) \) at time \( t \) is very small, then from (1.16) one gets

\[
h_T(t) = f_T(t) \quad (1.17)
\]

Furthermore, if one supposes the initial condition of \( F_T(0) = 0 \) or \( L_T(0) = 1 \), one will obtain from (1.13) to (1.16)

\[
L_T(t) = \exp \left[ - \int_0^t h_T(\tau) d\tau \right] \quad (1.18)
\]
Initial conditions different from zero are encountered in systems which are subject to periodic inspection. One may wish to calculate the reliability $L_T(t_1)$ at the time of inspection $t_1$, and then reliability $L_T(t_2)$ some time after the inspection, $t_2 > t_1$. If it is required to consider the time of periodic inspection as a new origin, one can define a variable $t' = t-t_1$. The initial conditions for this variable are then

$$F_{T'}(0) = F_T(t_1) \text{ and } L_{T'}(0) = L_T(t_1)$$

The reliability at time $t_2$ (or $t_2'$) is then obtained from

$$L_{T'}(t_2') = L_{T'}(0) \exp \left[ -\int_0^{t_2'} \eta_{T'}(\tau) d\tau \right] \quad (1.19)$$

One may treat "n" (number of load application) as a continuous random variable, and use Eq.(1.14) to (1.18). The error resulting from the discretization will be negligible in most practical applications.
2. A MODEL FOR CRACK INITIATION AND PROPAGATION

Failure occurs when the applied load effect exceeds the component resistance. Resistance of a structure diminishes in time due to either the repeated loading or wear process. Therefore, to determine the failure probability at a specified service life time, one has to be able to predict the residual strength of the component at the same instant. In the following a probabilistic fatigue crack growth model is developed.

2.1 Fatigue Life Model

Consider a component in service subjected to repeated loads applied from the service-load spectrum. There will be a period during which the process of load application leads to the formation of macroscopic cracks. The time (or number of load cycles) up to this stage will be termed service life to initial failure. The cracks will propagate as the life continues with a progressive reduction in the residual strength of the component. When a crack length reaches a critical level, \( a_F \), the crack propagation will accelerate and component fracture takes place after very few load applications. For practical purposes, one may consider the critical crack length is achieved when the residual strength is equal to the applied load effect.
Fig. 2.1: Evolution of a defect during service - lifetime of a population

Number of cycles

Microscopic zone

 Crack length

Median

Macroscopic zone

Critical level
Figure 2.1 shows a population of components varying from weak ones to those strong in resisting crack initiation and propagation. One notes in the figure three levels of crack length, microscopic, macroscopic, and critical (unstable). In Figure 2.1, the threshold crack length, $a_0$, could be envisaged as the crack level below which there is no reduction in strength as compared to the static loading.

At any given service life, $N_s$, there will be a variation in crack lengths in the population. Alternatively, a specified crack length will be obtained at various service life times in the population, (Fig. 2.1). These variations could be represented in a statistical form, i.e., as probability density functions. Figure 2.2(a) shows probability density function (PDF) of crack length at three different service life times. The PDF of fatigue lives for three different crack levels is shown in Figure 2.2(b).

Extensive test data surveys of similar components has shown that the fatigue life, $N_a$, for any given crack length, $a$, has a log-normal distribution with a variance $\sigma^2$ independent of the crack level[2,3]. The choice of distribution for fatigue life may be expressed as:

$$\log N_a = N(\log \bar{N}_a, \sigma^2_{\log N})$$

(2.1)
Fig. 2.2(a): Probability density function of the crack length at a specified lifetime

Fig. 2.2(b): Probability density function of the fatigue-life for a specified crack length
The two parameters of this distribution are the median fatigue life at the crack length \( a \), \( \tilde{N_a} \), and the variance \( \sigma^2_{\log N} \). The letter \( N \) in the front of the parenthesis in (2.1) means that the distribution of \( \log N_a \) follows a normal distribution. Recent tests carried out in our laboratory confirm the above distribution. A more general distribution function (three-parameter Weibull) was chosen to analyse damage and fracture in biaxial fatigue of thin-walled tubes [4]. An interesting characteristic of the Weibull distribution is that other distributions such as normal and log-normal, etc. are obtained as particular cases depending upon the choice of parameters. Unbiased analysis of experimental data show a tendency towards a log-normal type of distribution.

The choice of the log-normal distribution (2.1) then leads to the following corollary. For all points on the crack propagation curve for any component, the fatigue life has a constant ratio to the median life at the same crack length, i.e.,

\[
N_{a,z} = z\tilde{N}_a \quad (2.2)
\]

for all values of crack length \( a \). This relation is shown in Fig. 2.3 for a typical crack propagation curve of a component and the median curve of the population. It simply states that the time (or number of cycles) required for a crack to reach
Fig. 2.3: Life factor and crack propagation curves
level \( a \), is equal to the time (or number of cycles) for the median structure to reach the same crack length \( a \), multiplied by a factor \( Z \). In other words, for a given component or structure, the ratio \( N_a, z/\bar{N}_a \) is constant and independent of the crack length, Fig. 2.3. However, the value of this constant \( Z \) varies in the population from one component to the other, and is of a statistical nature. This is evident from relation (2.2) that for the median curve \( Z = 1 \). Therefore, all those to the left of the median curve (weak components) have a value \( Z < 1 \) and for all those to the right (strong components) \( Z > 1 \) (Fig. 2.1). The probability density function (PDF) of \( Z \), has a log-normal distribution for a given crack length. This is a consequence of the choice of distribution for the fatigue life (relation 2.1). The variance of the PDF of \( Z \) is also independent of the crack length, and the median \( \bar{m}_Z = 1 \).

The two relations (2.1) and (2.2) form the basis of the probabilistic model for fatigue life of a component. To characterize the life of a given component, it will suffice to have a knowledge of two variables: a) median curve of crack-propagation of the population, \( g(\bar{N}_a) \), and b) the distribution of parameter \( Z \) which is also called the "life factor".
2.2 Residual Strength of a Cracked Component

Structures (or components) with similar crack length may exhibit different residual strength. One may therefore assign a distribution for the residual strength $R(a)$, for a given crack length, $a$. It is reasonable to suppose that the mean value of residual strength $\mu(R)$ will be a decreasing function of crack length $a$, or time $T$, Fig. 2.4. This can be demonstrated by observing the macroscopic zone of Fig. 2.1. The crack length increases with the application of load cycles, consequently the residual strength will decrease. Test results in Refs. [2,4] clearly confirm this trend. A log-normal distribution similar to (2.1) may be considered for the residual strength distribution. Figure 2.4 shows schematically the mean residual strength and variation of it. For crack length $a_1$, the mean residual strength is $\mu_R(a_1)$, with a conditional probability density function $f_{R|A}(r,a_1)$. For another crack length $a_2 > a_1$, the mean value $\mu_R(a_2)$ will be less than $\mu_R(a_1)$ but with an identical conditional probability density function. This type of PDF is supported by a number of other works in reliability analyses see from example Refs. [5,6,7]. However, other types of distribution functions could easily be incorporated in the proposed model.
Fig. 2.4: Variation of the mean residual strength with crack length.
2.3 Variability of the Median Residual Strengths

The mean residual strength may be derived from the fracture mechanics principles. It has been pointed out that at a given service life, \( N_s \), one would find different crack lengths (see Fig. 2.2-a) in the population. Associated with these crack lengths, there will exist corresponding mean residual strength with their appropriate conditional PDF distribution about the mean. Figure 2.5 shows the residual strengths as a function of crack length and number of applied loads or service life. For example, at life cycle \( N_s \), one will find crack lengths, \( a_i, a_k \), etc..., and consequently, from Fig. 2.4, one would have mean residual strengths \( \mu_R(a_i), \mu_R(a_k), \) etc. Around the means, there would also be a conditional PDF, 
\[ f_{R|A,N}(r,a_i,N_s) \] and \[ f_{R|A,N}(r,a_k,N_s) \], respectively.

It should be noted that the above description does not introduce an additional variable to the formulation, it simply states the variability of the components' response to loading and residual strengths for a given crack length.

To summarize, at a given life cycle, \( N_s \), different crack lengths would be detected in the population (Fig. 2.1). For each of these crack lengths, \( a_i \), one would find a set of residual resistances, \( R(a_i) \), (Fig. 2.4) with a mean value and conditional distribution function, \( \mu_R(a) \) and \( f_{R|A}(r,a_i) \).
Fig. 2.5: Variation of mean residual strength with number of cycles or time.
This mean residual strength is a decreasing function of crack length and may be determined from fracture mechanics considerations. Finally, since there will be different crack lengths at a given service life $N_s$, one will find a corresponding number of mean residual strengths with an appropriate distribution about them (Fig. 2.5).

2.4 Mean Residual Strength of Pressurized Pipes Containing a Defect

The strength of a pressurized pipe containing a defect was determined in Phase I(a) of this work [8] based on fracture mechanics analysis and a large body of experimental data. The analysis will not be repeated here and the reader is to consult Ref. [8] for the detailed analysis. We would however, reproduce two figures indicating the relationships between the residual strength and crack length for thin and thick-walled pipes, Figs. 2.6 and 2.7, respectively. These curves can now be used in the proposed model to depict the mean residual strength schematically shown in Fig. 2.4.
Fig. 2.6: Residual strength versus crack length for a thin-walled pipe
Fig. 2.7: Residual strength versus crack length for a thick-walled pipe
3. PREDICTION OF FAILURE MODES

3.1 Possible Failure Modes

With the model developed in the previous section, it is now possible to derive failure risks. Figure 3.1 depicts three possible failure modes at a given life of any member of the population. The detectable crack length is denoted by $a_0$ (see also Fig. 2.1). Failure due to chance occurrence of a service load greater than the initial strength of the component will be termed static failure. It is to be noted that at defect length $a < a_0$, by definition, there is no reduction in the strength of the component. Thus, the probability of failure in this stage is independent of the defect size. However, as the crack (defect) length moves through the macroscopic zone, i.e., when $a_0 < a < a_F$, there will exist a greater probability of failure. The strength of the component is less than its initial value due to the existence of longer cracks. Failure is then due to the chance occurrence of a service load greater than the remaining strength (residual strength) of the component. This will be termed "static fracture due to fatigue", and it occurs when $a_0 < a < a_F$.

Finally, when the crack length reaches its critical value, $a_F$, it propagates rapidly and the residual strength takes a steep descent. Failure occurs when the residual strength
Fig. 3.1: Three modes of failure as function of crack length and number of cycles at $N_s$.
becomes less than the load effect. Figure 3.1 shows that at a life cycle $N_s$, there are a certain number of components which may fail in any of the three modes described above.

3.2 **Statistical Variables and Non-Dimensional Parameters**

There are three random variables in the proposed model. They are: the strength $R$, loading $S$, and crack (defect) length $A$. In addition, if one wishes to calculate the reliability or probability of failure in a specified time, then the number of cycles $N$, will also become a variable.

The probability of failure can then be calculated from (1.6) as follows:

$$P_f = \iiint_{R < S} f_{S,R,A,N}(s,r,a,n) \, dn \, dr \, da \, ds \quad (3.1)$$

Generally, the applied load $S$ is statistically independent from other variables. In such a case, (3.1) may be written as

$$P_f = \iiint_{R < S} f_S(s) f_{R,A,N}(r,a,n) \, dn \, dr \, da \, ds \quad (3.2)$$

If the probability of failure during a given life cycle is desired, one may use the notion of marginal joint probability density [9]. The probability of failure can then be calculated from:
It is to be noted that the integration in (3.3) is carried out over three variables instead of four as in (3.2). The result obtained however is a function of the fourth variable, \( N \).

To make the formulation more general, and to facilitate discussion of the results, the variables will be non-dimensionalized. For example, instead of referring to the service life of the component, one will talk about relative life \( V = N/N_i \) where \( N_i \) is the median of distribution of service life to first formation of detectable crack, sometimes called median of service life to initial failure, Fig. 3.1. The relative crack length is defined as \( H = a/a_p \) where \( a_p \) is the critical crack length. Finally, the relative residual strength \( X(a) = R(a)/\mu_R(a) \) where \( \mu_R(a) \) is the mean residual strength of a component containing crack length of size \( a \). The mean residual strength \( \mu_R(A) \) or \( \mu_R(H) \) could be written as

\[
\mu_R(H) = \mu_0 \cdot \phi(H) \tag{3.4}
\]

where \( \mu_0 \) is the mean strength of macroscopically un-cracked or initial structures (see Fig. 2.4) and \( \phi(H) \) is a decreasing function of the relative crack length, \( H \). It should be recalled that the residual strength of structures (or components) with
a defect size of $H$ have a distribution about the mean value $u_R(H)$ as depicted in Fig. 2.4.

The only mathematical operation made to arrive at the relative strength $X$, or relative crack length $H$, is division by a constant. Consequently, one may write [9]

$$f_X(x)dx = f_R(r) dr$$

$$f_H(h)dh = f_A(a) da$$

(3.5)

The probability of failure (3.3) in terms of non-dimensional variables is then

$$P_f(V_S) = \iint f_S(s)f_{X,H,V}(x,h,V_S)dhdxds$$

(3.6)

Using this general relation, one can now derive the risk and survival functions for the three failure modes shown in Fig. 3.1.

3.3 Static Fracture

It has been mentioned in section 2, that at the microscopic zone, the resistance of the structure is not affected by the existence of microscopic cracks, $a < a_0$. Therefore, by the definition of static failure, the relative residual strength $X$ and the relative crack length $H$, are statistically independent. Thus (3.6) can be written as
Defining the limits of integration, one gets

\[ P_{f_{sta}}(V_s) = \iiint_{R < S} f_S(s) f_x(x) f_{H,V}(h,V_s) dh dx ds \]  

(3.7)

or

\[
P_{f_{sta}}(V_s) = \int_{H=H_0}^{H=H_0} \int_{x=0}^{x=\infty} f_{x}(x) \left[ 1 - \int_{S=0}^{S=x, u_0} f_S(s) ds \right] f_{H,V}(h,V_s) dh dx
\]  

(3.8-a)

Evaluation of relation (3.8) may be somewhat difficult because it requires a knowledge about the distribution of \( f_{H,V}(h,V_s) \) at the microscopic zone. Recall that in the macroscopic zone, the distribution of crack lengths at a specified life is log-normal. This observation was based on the experimental evidence.

A similar procedure may prove difficult since measurement of microscopic crack lengths would be a very demanding task. There is however, an alternative way to resolve this problem. A variable change will now be carried out. Figure 3.2 shows a series of curves which are now familiar (Fig. 2.1) except that the variables are non-dimensionalized. One also notes that structures or components having a relative crack length \( H < H_0 \) at a service life \( N_s \) will all intersect the horizontal axis \( (H=H_0) \)
Fig. 3.2: Change of Variable for Static Fracture
to the right of $V = V_s$. These components or structures have therefore a relative initial fatigue life greater than $V_s$, i.e.

$$V_i \geq V_s \tag{3.9}$$

The relative initial fatigue life is defined by

$$V_i = \frac{N_i}{\hat{N}_i} \tag{3.10}$$

There exists a parallel between the above relation (3.10) and that of (2.2). In effect (3.10) may be written as

$$V_i = \frac{N_A = a_0 Z}{\hat{N}_A = a_0} = Z \tag{3.11}$$

The above relation effectively states that the initial life to fatigue is also the life factor $Z$. Thus (3.9) becomes

$$Z \geq V_s \tag{3.12}$$

It has been noted that the life factor $Z$ has a log-normal distribution. One may now interchange variables $H$ and $Z$ according to the following rule [9]:

$$f_H(h)dh = -f_Z(z)dz \tag{3.13}$$
The negative sign in Eq. (3.13) signifies that in life \( V_s \), the greater the \( H \), the smaller the \( Z \) and vice versa. In other words, the slope of the curve \( H \) vs \( Z \) at \( V_s \) is negative. Using (3.13) one has

\[
\int_{H=-\infty}^{H=H_0} f_{H,V}(h,V_s) \, dh = \int_{Z=V_s}^{Z=\infty} f_Z(z) \, dz \quad (3.14)
\]

and finally (3.8-a) becomes

\[
P_{f_{STA}}(V_s) = \left\{ \int_{X=0}^{X=\infty} f_X(x) \cdot [1-F_S(x,u_0)] \, dx \right\} \cdot \left[ 1-\int_{Z=0}^{Z=V_s} f_Z(z) \, dz \right]
\quad (3.15)
\]

The probability of static failure at \( V_s \) can now be calculated from the relation (3.15) containing three statistical variables \( X, S \) and \( Z \). The R.H.S. of Eq. (3.15) evaluates the probability of failure at \( V_s \) among those components which have survived up to \( V_s \). The probability thus is the risk of failure (see section 1, Eq. 1.13). Therefore,

\[
h_{STA}(V_s) = \left\{ \int_{X=0}^{X=\infty} f_X(x) \cdot [1-F_S(x,u_0)] \, dx \right\} \cdot \left[ 1-\int_{0}^{V_s} f_Z(z) \, dz \right]
\quad (3.16)
\]
The reliability can be calculated from relation (1.18) as follows:

\[ L_{STA}(V_s) = \exp \left[ -\int_0^{V_s} h_{STA}(\tau) d\tau \right] \tag{3.17} \]

where \( L_{STA}(V_s) \) is the reliability against static failure at life \( V_s \).

3.4 Static Fracture Initiated by Fatigue

This type of failure is due to the chance occurrence of a service load greater than the remaining strength of a structure, weakened by a crack which has not yet reached the stage of fatigue fracture. One could define a joint marginal density function on \( V \) of the variables \( X, H \) and \( V \), i.e., \( f_{X,H,V}(x,h,V_s) \).

One way of doing this is by using joint conditional density of \( X \) on \( H \) and marginal on \( V \), i.e., \( f_{X,H,V}(x|h,V_s) \). The Eq. (3.16) then becomes [9]:

\[ P_{f_{SFF}}(V_s) = \iint_{R < S} f_S(s) f_{X,H,V}(x|h,V_s) f_{H,V}(h,V_s) dhdxds \tag{3.18} \]

At the life \( N \), due to the variability in the fatigue property, the population of structures will contain cracks of varying degrees. However, those cracked to a given length \( A \) (or \( H \)) will have a variability in structural resistance about some mean value \( \mu_R(A) \) which is a function of \( A \) (see Fig. 2.4).
It has been pointed out that although the structural resistance depends on the crack length a (or H) the distribution of the relative strength \(X_f\), is the same for all values of crack length a. In particular, this distribution is the same as that of the relative strength of uncracked structures. In other words, although the fatigue cracking reduces the static strength of the structures, it does not affect the variability in the relative structural resistance. This is supported by some test results on the residual strength of structural components [8,10]. Therefore, the distribution about the mean is

\[
f_{x,H,V}(x|h,v)\,dx = f_x(x) \quad \text{with} \quad x = R/\mu_R(H)
\]

which leads to re-writing of (3.18) in the following form:

\[
P_{f_{SFF}}(v) = \int \int f_s(s)f_x(x)f_{H,V}(h,v)\,dh\,dx\,ds \quad (3.19)
\]

Now specifying the limits of integration results in:

\[
P_{f_{SFF}}(v) = \int_{H=H_0}^{H=H_1} f_{H,V}(h,v) \left\{ \int_{X=0}^{X=\infty} f_x(x) \cdot \left[ 1 - \int_{0}^{X} \mu_0 \phi(h) \, dx \right] \, dh \right\}
\]

or

\[
P_{f_{SFF}}(v) = \int_{H=H_0}^{H=1} f_{H,V}(h,v) \left\{ \int_{X=0}^{X=\infty} f_x(x) \cdot \left[ 1 - F_s(x,\mu_0,\phi(h)) \right] \, dx \right\} \, dh
\]

(3.20-b)
The probability of static fracture due to fatigue at a relative life \( V_s \), can be calculated from (3.20-b) in terms of statistical variables \( X, H \) and \( S \). As in the case of static failure, it is more convenient to express (3.20-b) in terms of \( X, Z \) and \( S \). The reason for this is due to the existence of considerable data on the variability in fatigue life.

The Eq. (3.13) already permits transformation from the variable \( H \) to \( Z \). The only other step required is to define the limits of integration of \( Z \) at a life \( V_s \), to obtain a relation similar to that of (3.14). Figure 3.3 shows the fatigue behaviour of three structures from the population: the median, the structure for which a crack length of \( H=1 \) is developed at the life of \( V_s \), and the other for which at \( V_s \), \( H=H_0 \). The relative life for the median structure \( V_i = \tilde{N}_i/\tilde{N}_i = 1 \) and the final life, \( V = \tilde{N}_f/\tilde{N}_i = \tilde{V}_f \). It has been shown that the relative life for a structure which has developed a crack length of \( H=H_0 \) at \( V_s \) is the life factor \( Z \). Using the relation (2.2) for a relative crack length of \( H=1 \), one gets:

\[
Z = \frac{N_{A=a_F,Z}}{\tilde{N}_{A=a_F}} = \frac{V_s \cdot \tilde{N}_i}{\tilde{V}_f \cdot \tilde{N}_i} = \frac{V_s}{\tilde{V}_f} \tag{3.21}
\]

Thus the quantity \( V_s/\tilde{V}_f \) represents the point of intersection of the fatigue response curve of the structure with the axes \( H=H_0 \), Fig. 3.3.
The limits of integration are, therefore:

\[
\int_{H=H_0}^{H=1} f_{H,V}(h,v) \, dh = \int_{V_s/V_f}^{V_s} f_z(z) \, dz \quad (3.22)
\]

The equation for the median crack-propagation curve has the following form (Fig. 3.3)

Fig. 3.3: Bounds of Integration for Static Fracture Due to Fatigue
From relation (2.2),
\[ \tilde{V}_H \tilde{N}_i = \frac{V_H \tilde{N}_i}{Z} \]

which when substituted in (3.23), yields:
\[ H = g\left(\frac{V_H}{Z}\right) \] (3.24)

From the above relation the crack length at any period of life can be determined if the life factor \( Z \) and the median crack propagation curve are known. The Eq.(3.20-b) can now be written in terms of the three variables \( X, Z \) and \( S \):

\[
P_{\text{SFF}}^{(V_s)} = \int_{Z=V_s}^{Z=V_s} f_Z(z) \left\{ \int_0^\infty f_X(x) \left[ 1 - F_S(x, \mu_0 \phi(g(V_s/Z))) \right] dx \right\} dz
\]

\[
dx \ \ dz \quad (3.25-a)
\]

Since relation (3.25-a) evaluates the probability of failure among structures which have survived up to \( V_s \), it is therefore by definition (Eq.1.13) the risk of failure, i.e.

\[
h_{\text{SFF}}(V_s) = \int_{V_s/\tilde{V}_f}^{V_s} f_Z(z) \left\{ \int_0^\infty f_X(x) \left[ 1 - F_S(x, \mu_0 \phi(g(V_s/Z))) \right] dx \right\} dz
\]

\[
(3.25-b)
\]
The corresponding probability of survival (or reliability for static fracture due to fatigue) can be calculated from:

$$L_{SFP}(V_s) = \exp \left(-\int_0^{V_s} h_{SFP}(\tau) d\tau\right) \quad (3.26)$$

### 3.5 Fatigue Fracture

Referring to Fig. 3.1, one notes that at a given life, a portion of the population will have cracked to a crack length $a_F$, and will therefore have failed by complete fatigue fracture. The fatigue fracture can be viewed as a particular case of static fracture by fatigue when the crack length has reached the critical length $a = a_F$. Therefore, Eq. (3.19) can be used to evaluate the probability of failure at $V_s$. In cases where a constantly applied load is carried by the structure, $a_F$ becomes the crack length to cause collapse under this applied static load. In such a situation the probability of failure of a certain part of Eq. (3.19) becomes:

$$\int \int_{R < S} f_S(s)f_X(x) dx ds = 1 \quad (3.27)$$

The remaining part is then:

$$P_{f_{FF}}(V_s) = \int_{H=1}^{H=\infty} f_{H,V}(h,V_s) dh \quad (3.28)$$
Applying the relation (3.13) and defining the limits of integration in terms of \( Z \), one can write Eq.(3.28) such that the integration is carried out in terms of \( Z \). For \( \bar{h}=1 \), it has been shown that \( Z=V_s/\tilde{V}_f \) and for structures which have cracked to the extent that \( \bar{h}=\infty, Z=0 \). Thus Eq.(3.28) takes the form of:

\[
F_{PF}(V_s) = \int_0^{V_s/\tilde{V}_f} f_Z(z)\,dz \quad (3.29-b)
\]

This equation determines the probability of fatigue fracture at \( N_s \) (or \( V_s \)) for all structures in the population which have developed a relative crack length of \( \bar{h}=1 \). It takes account of all structures which could have fractured at life cycles \( N_s-1, N_s-2, \ldots, \) etc. It therefore corresponds to the definition of the cumulative density function (CDF) defined by the relation (1.14). Thus we may write

\[
F_{PF}(V_s) = \int_0^{V_s/\tilde{V}_f} f_Z(z)\,dz \quad (3.29-b)
\]

Using (1.16), the risk of fatigue fracture would be

\[
h_{PF}(V_s) = \frac{f_Z(V_s/\tilde{V}_f)}{1 - F_{PF}(V_s)} \quad (3.30)
\]
5. FUNCTIONAL RELATIONSHIPS BETWEEN RELATIVE LIFE OR RELATIVE STRENGTH AND RELATIVE DEFECT SIZE

The relationship between the crack propagation curve versus life cycle was derived in the Phase I-a report. Figure 5.1 reproduced here shows the number of cycles versus crack length for a specified stress level. It is to be recalled that the pipe material was A-212B low carbon steel at 600° F [11].

The relative life is defined by

\[
V = \frac{N + \tilde{N}_i}{\tilde{N}_i} = \frac{\tilde{N}}{\tilde{N}_i} \tag{5.1}
\]

where \(N\) is the number of applied cycles excluding those which were required to initiate a fatigue crack. \(\tilde{N}_i\) is the median of the distribution of initial cycles \(N_i\) to form a fatigue crack.

The relative crack length is expressed by

\[
h = \frac{2a}{2a_f} \tag{5.2}
\]

where \(a_f\) is the critical crack length, defined earlier. The relative life and crack length corresponding to Fig. 5.1 are given in Table 5.1. Functional relationship may be found by the least-square technique. The resulting curve is depicted in Fig. 5.2. To accurately represent the results, the Table 5.1 values were divided into three parts and least-square curves were fitted for each part.
determined a priori and high failure rates in the early life may alter the shape of the assumed distribution. This however, is not of concern in a practical situation.
4. EFFECTS OF INSPECTION

The effect on the risk of failure (or reliability) of various inspection procedures in service can now be investigated by using the expressions derived in the previous section.

4.1 Continuous Inspection

In a continuous inspection as soon as cracks reach the relative detectable length $H_d$, corresponding to a life $\tilde{V}_d$ on the median crack propagation curve, the structure or component is removed from service or repaired to its original state. In this type of inspection the fatigue fracture is prevented as long as $H_d < 1$. Fig. 4.1 shows the effect of such a procedure on the crack propagation curves. Another effect of the continuous inspection is reduction of cracked structures in the population. This will then reduce the risk of static fracture due to fatigue. In practice, when cracks are detected they are usually repaired and the structure or component is restored to service. In this event there is no depletion of the population by the inspection process. The risk of static fracture by fatigue can be obtained directly from Eq. (3.20-b).
Fig. 4.1: Effect of $H_d$ on the Defect Growth
The only change is the replacement of the upper limit of integration of relative crack length to $H = H_d$ in case of continuous inspection. When there is no inspection, this limit is $H = 1$.

Interchanging the variables in the same manner as that from Eq. (3.20) to (3.25), one may write Eq. (4.1) in terms of life factor $Z$. Referring to Fig. 4.1, it can be seen that this corresponds to integration from $Z = V_s/\tilde{V}_d$ to $Z = V_s$, and hence

\[
\begin{align*}
    h_{SFF}^{CON}(V_s) &= \int_{V_s/\tilde{V}_d}^{V_s} f_{Z}(z) \left\{ \int_{0}^{\infty} f_X(x) [1-F_S(x, \mu_0 \phi(h))] dx \right\} dz \\
    &= \int_{V_s/\tilde{V}_d}^{V_s} f_{Z}(z) \left\{ \int_{0}^{\infty} f_X(x) [1-F_S(x, \mu_0 \phi(g(V_s/z))] dx \right\} dz \\
\end{align*}
\]

(4.2)

where $\tilde{V}_d$ is the relative life of the median structure to reach a relative detectable crack length of $H_d$.

As for the static failure, it is not affected by the inspection as long as $H_d \geq H_0$. In this case the number of structures or components having microscopic cracks will not change, therefore, the risk of static failure is obtained from the Eq.(3.16).

The total risk of failure is then the sum of the two, i.e.

\[
    h_{TOT}^{CON}(V_s) = h_{STA}(V_s) + h_{SFF}^{CON}(V_s)
\]

(4.3)

The probability of survival or reliability is calculated from relation (1.18).
4.2 Periodic Inspection

In practice, it is not feasible to inspect components or systems continually, but inspections are carried out periodically at intervals $V_I(1)$, $V_I(2)$, etc. Until the first inspection at relative life of $V_I(1)$, the conditions in the population are the same as for no inspection. Cracks initiate and grow undetected, and risks of failure can be determined from Eqs. (3.16), (3.25) and (3.30). Figure 4.2 shows what happens when an inspection is made at life $V_I(1)$. All components with cracks exceeding $H_d$ are removed. The effect of such an operation is to eliminate the risk of fatigue rupture for a certain time. The time interval is simply that which would require a crack length of $H=H_d$ at time $V_I(1)$ to propagate to $H=1$. The question is to find the time required for the cracks to reach the critical length. Once again the notion of life factor (Eq. 2.2) will be used to determine bounds of integration. The relative initial fatigue life of a structure (or component) with a crack length $H_d$ (Fig. 4.2) at $V_I(1)$ (Fig. 4.2) is:

$$V_i = Z = V_I(1)/\bar{V}_d$$

The time required for the crack to propagate to $H=1$, may be determined from:

$$N_a, Z = Z \bar{N}_f = \frac{V_I(1)}{\bar{V}_d} \cdot \bar{N}_f$$ (4.4)
Fig. 4.2: Effect of Periodic Inspection on the Population of Structures with Various Defect Sizes.
Dividing both sides by the median initial life $\tilde{N}_i$, one gets the relative life values:

$$v_{a_r', z} = \frac{V_{I(1)}}{\tilde{V}_d} \cdot \tilde{V}_f$$  \hspace{1cm} (4.5)

This value is indicated on Fig. 4.2 as well as in the region where the risk of fatigue fracture is eliminated due to the inspection. However, once the upper limit of the interval (Eq. 4.5) is exceeded, there would be a risk of failure by fatigue fracture. To obtain this risk for the life $v_s > V_{I(1)} \frac{\tilde{V}_f}{\tilde{V}_d}$, and $v_s / \tilde{V}_f$, i.e.

$$h_{PF}^{PER}(v_s) = \frac{f_z(v_s / \tilde{V}_f)}{1 - \int_{V_{I(1)}/\tilde{V}_d}^{V_s / \tilde{V}_f} f_z(z) dz}$$  \hspace{1cm} (4.6)

At the instant of inspection all crack lengths are reduced to a value $H < H_d$. The risk of static failure due to fatigue then drops from that for no inspection, Eq. (3.25) to that of continuous inspection, Eq. (4.2). After the periodic inspection at $V_{I(1)}$, the cracks may propagate freely. If one wishes to calculate the risk of static failure due to fatigue, two cases may arise:

(i) $V_{I(1)} \leq v_s \leq V_{I(1)} \frac{\tilde{V}_f}{\tilde{V}_d}$

(ii) $v_s > V_{I(1)} \frac{\tilde{V}_f}{\tilde{V}_d}$
In the first case all relative crack lengths are less than one $H_0 < H < 1$ and risk of failure is obtained from Eq. (4.25-b) with appropriate substitution for integration limits:

$$h_{SFF}^{PER}(V_s) = \int_{V_I(l)/\tilde{V}_d}^{V_s} f_z(z) \left\{ \int_0^\infty f_X(x)[1-F_S(x, u_0, \Phi [g(\frac{V_s}{z})])] \, dx \right\} \, dz$$

(4.7)

For the second case there will be relative crack lengths ranging from 0 to 1, which correspond to the no-inspection case. The risk of static fracture due to fatigue is then obtained from Eq. (2.5).

The risk of static failure is not affected by the periodic inspection since $H_d > H_0$.

To summarize, in a periodic inspection two cases are encountered: a) when $V_s < V_I(l) \tilde{V}_f/\tilde{V}_d$, there will exist a chance occurrence of static failure and static rupture due to fatigue. In this case one would use Eq. (3.16) and (4.7) to determine the appropriate risks; b) when $V_s \geq V_I(l) \tilde{V}_f/\tilde{V}_d$, the risk of fatigue failure will be present, and it may be evaluated from Eq. (3.30). As for the other failure modes, they are the same as the no-inspection case, and Eqs. (3.16) and (3.25) may be employed to determine their risk value. The total risk of failure in each case is determined from:
Finally, an interesting point is the probability of detecting the crack or defect of a given size during the periodic inspection. From Fig. 4.2, it is clear that the crack lengths to be detected at $V_I(l)$ are between $H_d$ and 1. The probability of detecting the cracks of length $H > H_d$ in those structures of the population is

$$h_{DET}(V_I(l)) = \int_{V_I(l)/\tilde{V}_d}^{V_I(l)/\tilde{V}_f} f_z(z)dz \quad (4.10)$$

If it is required to determine the probability of crack detection at the $m^{th}$ periodic inspection at a relative life of $V_I(m)$, two cases have to be studied:

1) $H_{MAX} < 1$ for $V_I(m) < V_I(m-1) \cdot \tilde{V}_f/\tilde{V}_d$

2) $H_{MAX} = 1$ for $V_I(m) \geq V_I(m-1) \cdot \tilde{V}_f/\tilde{V}_d$

In the first case, the probability of crack detection is given by:
In this section relationships have been derived allowing determination of probability of survival or reliability of structures or components based on a model developed in Section 2. An essential feature of the proposed model is knowledge of crack propagation and residual strength of the median of the population. The residual strength versus crack length of pressurized pipe was determined in the Phase I(a) report [8] and is reproduced in Figs. 2.6 and 2.7 in Section 2 of this report.

In the following, functional relationships between relative life, relative crack length and relative residual strength will be developed.
5. FUNCTIONAL RELATIONSHIPS BETWEEN RELATIVE LIFE OR RELATIVE STRENGTH AND RELATIVE DEFECT SIZE

The relationship between the crack propagation curve versus life cycle was derived in the Phase I-a report. Figure 5.1 reproduced here shows the number of cycles versus crack length for a specified stress level. It is to be recalled that the pipe material was A-212B low carbon steel at 600°F [11].

The relative life is defined by

\[ V = \frac{N + \tilde{N}_i}{\tilde{N}_i} = \frac{\tilde{N}}{\tilde{N}_i} \]  (5.1)

where \( N \) is the number of applied cycles excluding those which were required to initiate a fatigue crack. \( \tilde{N}_i \) is the median of the distribution of initial cycles \( N_i \) to form a fatigue crack.

The relative crack length is expressed by

\[ h = \frac{2a}{2a_f} \]  (5.2)

where \( a_f \) is the critical crack length, defined earlier. The relative life and crack length corresponding to Fig. 5.1 are given in Table 5.1. Functional relationship may be found by the least-square technique. The resulting curve is depicted in Fig. 5.2. To accurately represent the results, the Table 5.1 values were divided into three parts and least-square curves were fitted for each part.
The relative residual strength has been defined earlier in the following form

\[ \phi(h) = \frac{\mu_R(h)}{\mu_0} \]  

(5.3)

where \( \mu_R(h) \) is the median residual strength of structures containing a relative crack length of \( h \), and \( \mu_0 \) is the median strength of uncracked structures. Table 5.2 gives the non-dimensionalized results corresponding to Fig. 2.7. Here again the least-square fits were used to obtain Fig. 5.3 showing the variation of relative strength versus relative life.
Fig. 5.1: Crack propagation vs number of cycles ($\sigma = 28,500$ psi)
### TABLE 5.1

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Stress level of 28,500 psi
Relative life and relative crack length for a
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**TABLE 5.2**

Relative life and relative residual strength for an applied stress of 28,500 psi
Fig. 5.2: Variation of relative crack length versus relative life
Fig. 5.3: Variation of relative residual strength versus relative life
6. SUMMARY AND CONCLUSIONS

A model is presented taking into account the existence of undetected defects and their statistical nature. The propagation of defects in time and the variability of material response and applied load spectrum is included in the proposed probabilistic model. A rigorous formulation is presented to evaluate the risk of various failure modes at a given time, and the effects of different types of inspection procedures on these risks. To determine the failure risks, or probability of survival at any given time, one would have to specify the following data:

(1) The probability density function of relative residual strength \( f_x(x) \) in terms of relative crack length \( H \).
(2) The probability density function of life factor \( f_Z(z) \).
(3) The median static value of the ultimate strength of material \( \mu_0 \).
(4) The variation of median residual strength \( \phi(h) \) in terms of relative crack length.
(5) The crack propagation curve in terms of number of cycles (or time) of the median of population \( H_Y = g(\tilde{V}_H) \).
The probability density function of loading (or stress) $f_s(s)$.

The limits of integration as determined by the type of inspection procedure employed.

The type of probability density function of residual strength $f_X(x)$ and life factor $f_Z(z)$ were described in section 2 of this report. A log-normal distribution is supported by test results. For a given material, items 3 and 5 can be determined from the experimental data. An example of this was given in Phase I-a report and is reproduced here in Fig. 5.2 for a nuclear piping steel (A-212B) at 600° F. The variation of relative residual strength in terms of relative life (item 4) for a pressurized pipe is given in Table 5.2 and Fig. 5.3.

It is to be noted that items 3 to 5 are dependent on the material properties and mean operational stress level. The examples given in section 5 are for a specific geometry of pipe and material. For any other material, one has to find either in the literature or by conducting a limited number of experiments, the required basic data.

In conclusion, this phase of investigation has produced a general formulation and has demonstrated how the data
could be obtained and applied to a specific case. The probabilistic formulation phase being completed, one needs only to develop computer programmes to evaluate the risk of different failure modes, effects of inspection on these risk values, or reliability of the component in time. One may also undertake a sensitivity analysis to evaluate the effects of inspection, defect detection method, etc.
7. REFERENCES


Periodic inspection serves two purposes. It is a very important aspect of preventive maintenance to protect the productivity of the plant, and it can, in some cases, make an important contribution to the safety of plant personnel and the public. Periodic inspection for safety has for a long time been an integral part of the licensing process for conventional boilers and pressure vessels, but in a rather crude way looking for gross effects such as massive corrosion or tube blockage and requiring a complete shut down of the plant for what has been mostly a visual inspection.

The monetary penalties resulting from the total shut down of process plants for inspection purposes becomes less and less tolerable as the unit size increases. To relieve this situation, procedures to monitor identified causes of deterioration, and the basing of the inspection interval on a running analysis of measured critical criteria are increasing. These procedures require that a close connection be achieved between the design of the plant and its performance. In aviation, periodic inspection was at first considered in the context of a preset safe life. This was to be carried out at intervals determined by a programme of testing and of major overhauls which could be modified in time by evidence coming from the inspection itself. With increasing fleet, unit size, and service frequency, periodic inspection is in many cases...
carried out at intervals determined by analysis of data from a line performance monitoring.

For nuclear reactors, the hostile radiation environment presents serious problems for re-inspection, requiring specially developed equipment as well as the lengthening of time required to carry out the inspection. To date the light water type has set the pattern for periodic inspection of reactors. The special problems of the large thick walled pressure vessel has caused great emphasis to be placed on the danger of catastrophic failure of the pressure envelope.

It is important to remember that the cyclic refuelling of light water reactors provides an opportunity for periodic inspection which does not exist with the continuous on power Candu type refuelling. For Candu reactors it is very important then to give careful attention to the requirement for periodic inspection because an overspecification will have a serious impact on production as well as on accumulated radiation doses.

The purpose of this study is to provide a rational approach to periodic inspection based on a probabilistic model.

8.1 Periodic Inspection for Safety

Periodic inspection for safety is intended to maintain an adequate level of safety throughout the life of the plant. In the case of nuclear reactors a defence in depth
is supposedly used against the potential hazard of the release of the fission product inventory of the core. This defence in depth takes the form of multiple barriers with the additional protection of "safety" systems which are intended to mitigate against the effect of the failure of an individual barrier; for example, emergency core cooling mitigates against the effect of the loss of normal coolant from the core which would result if a rupture of the pressurized reactor coolant system envelope occurred. There has been a requirement associated with the use of nuclear reactors that the residual risk must be reduced to the greatest extent possible. This is the situation for both routine (non-accidental) and accident conditions. This has resulted in the adoption of a postulatory approach to satisfy in which an upper limit accidental event is described for which the effects must be minimized within prescribed regulatory limits.

The problem of establishing the amount and frequency of re-inspection in practical terms has been a concern of the aviation industry since its inception. However, to survive in aviation it is necessary to learn to operate on the brink of a catastrophe, and consequently the safety margins are therefore small and very dependent on re-inspection procedures. In nuclear matters in general the margins should be much greater and there need be less reliance on re-inspection to maintain a satisfactory safety level throughout the life of the plant.
The ASME Section XI Code presently deals only with the reactor coolant pressure boundary. The code then requires partial re-inspection at each refuelling outage (12-18 months) to achieve 100% re-inspection in each 10 year interval.

Proposals so far made for a Candu Reactor Code also deal with the reactor pressure coolant boundary and certain interfaces with the secondary side (for example to steam generators), but classifies re-inspection requirements according to the effect a component failure would have on the safety of people and the likelihood of the event. For example re-inspection for safety might be limited to components the failure of which would initiate the emergency cooling system, since other events would not seriously tax the overall containment system. The Canadian plan is in the development stage and this study, in part, is intended to help in this process.

Explicit in the Canadian approach is the postulate of a component failure. One may then ask what is the process of such a failure and what is the probability of its occurrence?